Certifying Parity Reasoning Efficiently Using Pseudo-Boolean Proofs

Stephan Gocht, Jakob Nordström

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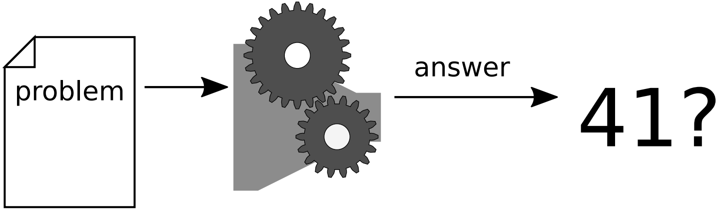
Detecting Bugs with Certifying Algorithms



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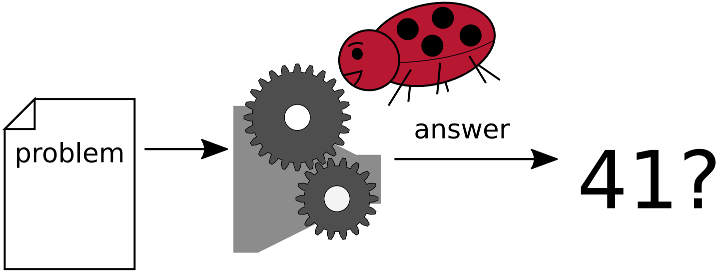

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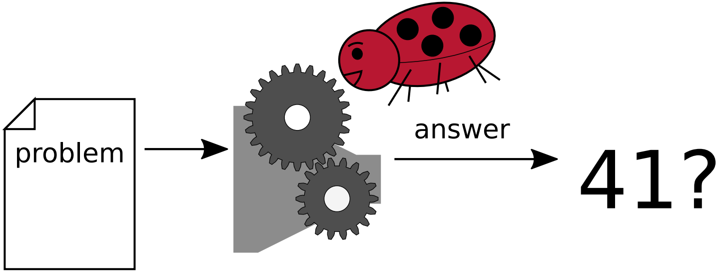

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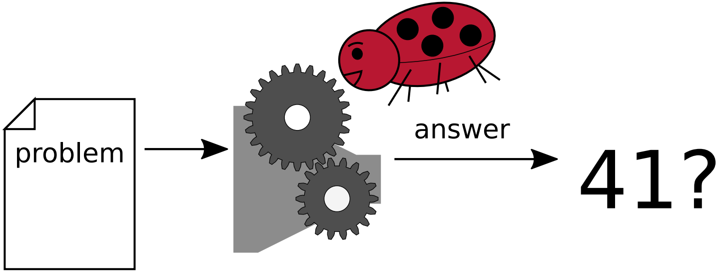

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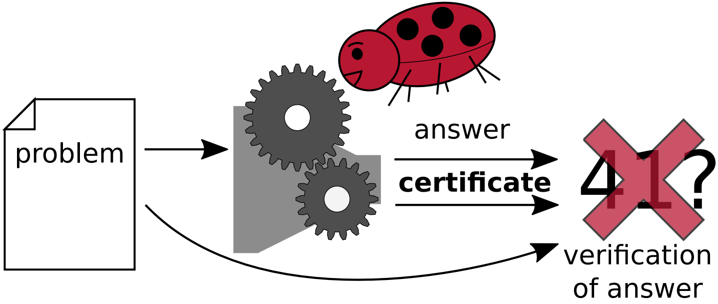

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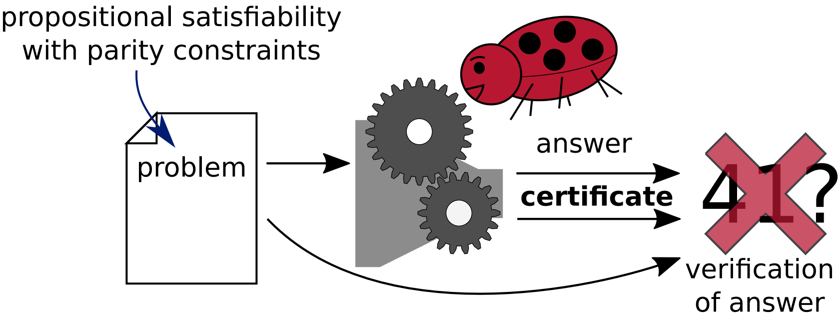

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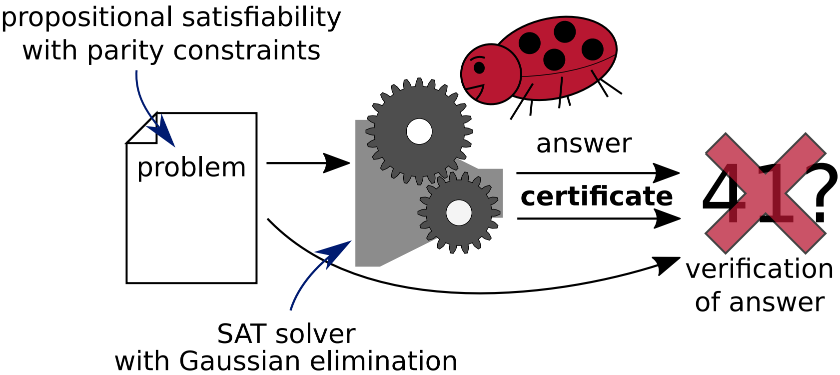

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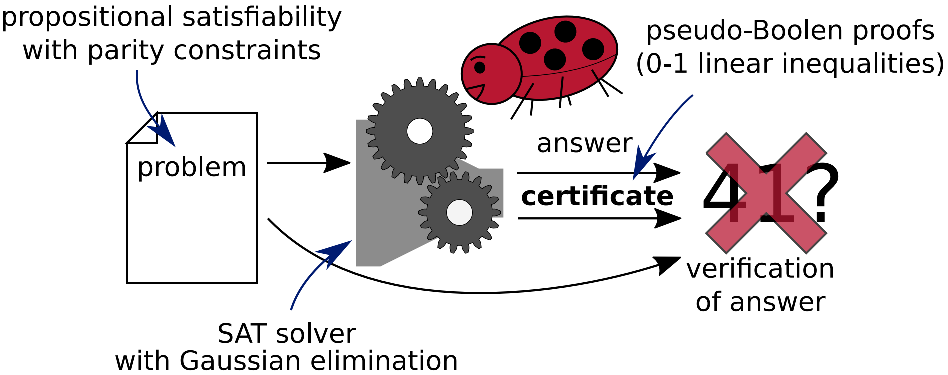

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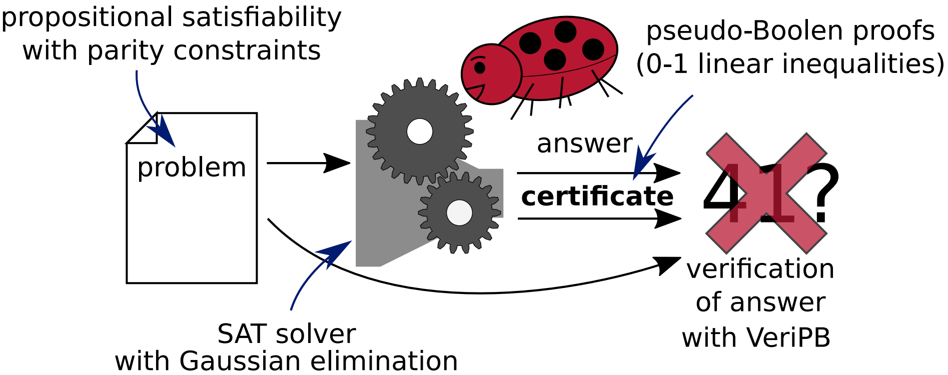

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SAT Solving — A Success Story for Certifying Algorithms . . .

- SAT = satisfiability testing of propositional formulas
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- Proof formats such as RUP [GN03], TraceCheck [Bie06], GRIT [CFMSSK17], LRAT [CFHH+17]; DRAT [WHH14] has become standard.
- certificates can help to
  - prove correctness of answer
  - detect and fix bugs, even when solver produced correct answer
  - audit answer later on
  - explain what solver is doing

...Except for SAT Solving Techniques That Can't Be Certified

- too much overhead / too complicated proof logging for
  - **Parity reasoning** (as in CryptoMiniSat [Cry] and Lingeling [Lin])
  - Counting arguments (as in Lingeling)
  - Symmetry breaking (as in BreakID [Bre])

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- How about practical proof logging for stronger solving paradigms?
  - MaxSAT solving
  - constraint programming (CP)
  - mixed integer programming (MIP)
  - algebraic reasoning / Gröbner basis computations
  - pseudo-Boolean satisfiablity and optimization

New Proof Systems on the Rise

many new proof systems with implemented proof checkers:

- propagation redundancy (PR) [HKB17a]
- practical polynomial calculus (PAC) [RBK18, KFB20]
- propagation redundancy for BDDs [BB21]
- Max-SAT resolution [PCH21]
- **pseudo-Boolean proofs** [EGMN20, GN21]

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applications:

- solving cryptographic problems
- approximate counting
- circuit verification

$$x_1 + x_2 + x_3 \geq 1$$
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- Boolean variable x with domain 0 (false) or 1 (true)
- Literal: x or its negation $\bar{x} = 1 - x$
- Pseudo-Boolean constraint:
  linear (in-)equality over literals
- Clause: at-least-one constraint
- Parity / XOR: equality modulo 2
  notation: $x_1 \oplus x_2 \oplus x_3 = 1$
- Assignment: function mapping variables to $\{0, 1\}$
- VeriPB Proof Format (PBP):
  - based on pseudo-Boolean constraints
  - has operations to reason with PB constraints

Goal: find assignment satisfying all constraints

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How can we formalize this?

Example

Step 1: Translate XORs

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All steps easily expressible in VeriPB!

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- no guarantee that all propagations detected
- SAT inprocessing requires to generalize DRAT (next slides)

More Notation

- (partial) substitution $\omega = \{ y_1 \mapsto 0 \}$
  function that maps variables to literals or $\{ 0, 1 \}$
- variable substitution

$$(x_1 + x_2 + x_3 \geq 2y_1)_{\restriction\omega} = x_1 + x_2 + x_3 \geq 0$$

- $F \models F'$: satisfying assignment to F is also satisfying assignment to F'

Substitution Redundancy Rule (Generalizing DRAT)

Substitution Redundancy (generalizing [HKB17b, BT19] to pseudo-Boolean)

Can add constraint C to formula F if and only if there is a *witnessing* partial substitution ω such that

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- generalizes DRAT [HKB17b]
- $\Rightarrow$ all SAT pre- and inprocessing techniques covered

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For fresh variable y_1 (not appearing in F), want to add...

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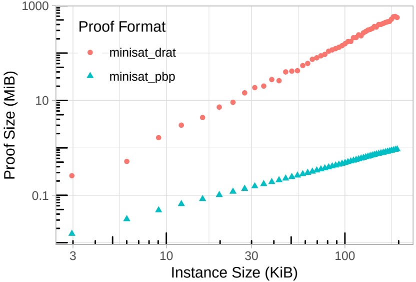
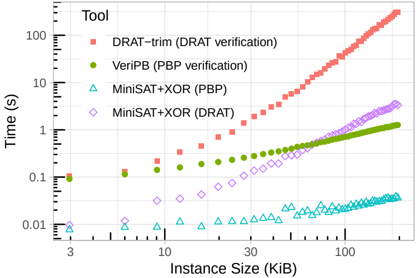
concrete proof format:

```
red  1 x1 +1 x2 +1 x3 -2 y1 >=  0 ; y1 -> 0
```

Experiments

- Implemented "plug and play" XorEngine with proof logging[1] in MiniSAT[2]
- Evaluated on crafted benchmarks (Tseitin-Formulas)
  represent worst case with single large XOR matrix
- DRAT proof for comparison [PR16]





[1] https://gitlab.com/MIAOresearch/xorengine
[2] https://gitlab.com/MIAOresearch/minisat_with_xorengine

Conclusion

- proof logging is well-established standard for SAT solving
- so far, prohibitively expensive for some techniques (XOR reasoning, counting arguments, symmetry breaking)

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Our work: Proof logging for SAT solving and XOR reasoning with VeriPB[3]

- simple to implement + efficient proof checking

[3] `https://gitlab.com/MIAOresearch/VeriPB`

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Our work: Proof logging for SAT solving and XOR reasoning with VeriPB[3]

- simple to implement + efficient proof checking

Future work:

- capture more types of reasoning within SAT solvers
  - counting arguments (should be straightforward)
  - symmetry breaking
- provide efficient proof logging also for other paradigms (MaxSAT, pseudo-Boolean optimization, MIP)
- new expressive proof formats and verifiers for competitions (why not with VeriPB ;-))

[3] `https://gitlab.com/MIAOresearch/VeriPB`

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