QuBE 7.0

Solver Description

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Pragmatics of SAT, Edinburgh, 2010 July 10th

Quantified Boolean Formulas

- Generalization of propositional logic
- Adds the *quantifiers* to the propositional variables
- Prototypical P–Space Complete problem

$$\varphi = \forall x(\exists y((x \vee y) \wedge \exists z(\neg x \vee y \vee z)))$$

closed QBF in Prenex Conjunctive Normal Form

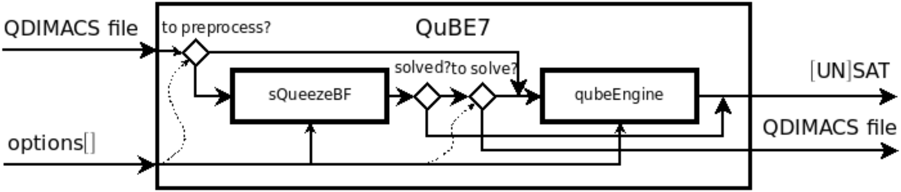
$$\overbrace{Q_1 z_1 \cdots Q_n z_n}^{\text{prefix}} \; \overbrace{\phi(z_1, \ldots, z_n)}^{\text{matrix}} \qquad n \geq 0$$

- Every Q_i ($1 \leq i \leq n$) is a quantifier, either existential $\exists$ or universal $\forall$
- Every z_i is a Boolean variable
- The level of a variable z_i with $j \geq i$ and $Q_j \neq Q_{j+1}$ is the number of alternating quantifier blocks from left to right (starting with 1)
- ϕ is a Boolean formula over the set of variables $\{z_1, \ldots z_n\}$ using standard Boolean connectives and the constants $\perp$ and $\top$

Outline

1. The Tool
2. sQueezeBF
   - Equivalence Reasoning
   - Variable Elimination by Q-Resolution
3. qubeEngine
   - Data Structure
   - Main Algorithms
   - Conflict and solution analysis

QuBE7.0 Architecture



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sQueezeBF

Why Preprocessing?

- QBF is a powerful extension/generalization of SAT
- for SAT formulae has been proven to be very effective, reducing size, extracting structural info, decreasing the total solving time
- it works very well for QBF too!
- to extend from SAT to QBF is not a trivial task!

GOALS

- Decrease as much as possible the size of the formula
- Explicitate hidden information

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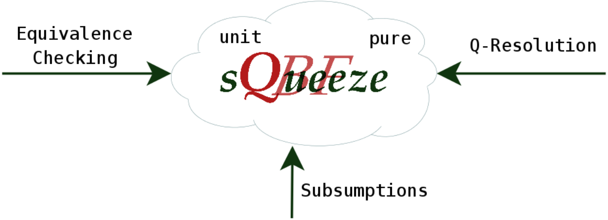

Preprocessing Loop

function *sQueezeBF*(φ)
  do
    φ' = φ
    φ = *Simplify*(φ)
    φ = *EquivalenceSubstitution*(φ)
    φ = *EquivalenceRewriting*(φ)
    φ = *Q-resolution*(φ)
    if $\varphi \equiv$ TRUE **return** φ
    if $\varphi \equiv$ FALSE **return** φ
  while $\varphi' \neq \varphi$
  return φ

Equivalence Reasoning

Real World Problems contain lots of equivalences:

$$l \equiv l_1 \vee l_2 \quad \Rightarrow \quad (\bar{l} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2})$$

$$l \equiv l_1 \wedge l_2 \quad \Rightarrow \quad (l \vee \overline{l_1} \vee \overline{l_2}) \wedge (\bar{l} \vee l_1) \wedge (\bar{l} \vee l_2)$$

$$l \equiv \overline{l_1} \quad \Rightarrow \quad (l \vee l_1) \wedge (\bar{l} \vee \overline{l_1})$$

$$l \equiv l_1 \equiv l_2 \quad \Rightarrow \quad (l \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1} \vee \overline{l_2}) \wedge (\bar{l} \vee l_1 \vee \overline{l_2}) \wedge (\bar{l} \vee \overline{l_1} \vee l_2)$$

$$l \equiv \gamma \quad \Rightarrow \quad (l \rightarrow \gamma) \wedge (\gamma \rightarrow l)$$

Equivalence Reasoning (cont'd)

The algorithm works in 2 steps:

1. identification of definitions
2. variable substitution

The size of the formula should not increase in terms of literals

$$dim(\varphi(l)) + dim(l \equiv l_1 \vee l_2) \leq \varphi(l_1 \vee l_2/l) + K$$

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Equivalence Reasoning (cont'd)

The algorithm works in 2 steps:

1. identification of definitions
2. variable substitution according to an order

If the size of the formula increases?

$$dim(\varphi(l)) + dim(l \equiv l_1 \vee l_2) > \varphi(l_1 \vee l_2 / l) + K$$

$\Rightarrow$ Equivalence Rewriting

Equivalence Rewriting

Transforms the given QBF

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi$$

into the equivalent

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge (\bar{l} \vee \overline{l'}) \wedge \phi$$

where:

$$level(l') = level(l)$$

Q-Resolution

- Given two clauses
  $C_1 = x \vee y_1 \vee ... \vee y_n$; $C_2 = \overline{x} \vee z_1 \vee ... \vee z_m$
  then the q-resolution between C_1 and C_2 over the variable $|x|$ is
  $$C = C_1 \otimes C_2 = y_1 \vee ... \vee y_n \vee z_1 \vee ... \vee z_m$$
- we can think a formula as two distinct sets :
  $\phi = \langle P(X_1, ..., X_n), M(X_1, ..., X_n) \rangle$ where :
  - $P(X_1, ..., X_n)$ is an ordered set of atoms
  - $M(X_1, ..., X_n)$ is a set of clauses over the variables $X_1 ... X_n$
- given a clause C then we define dim(C) = $|C|$
- given a set of clauses S, we define dim(S) = $\sum_{i=1}^{|S|} dim(C_i)$

Variable Elimination by Q-Resolution

Given

- $\phi = \langle P(X_1, ...X_n), M(X_1, ..., X_n)\rangle$
- $S_x \subseteq M : \nexists S'_x \subseteq M$ such that $S'_x \not\subseteq S_x$
- $S_{\overline{x}} \subseteq M : \nexists S'_{\overline{x}} \subseteq M$ such that $S'_{\overline{x}} \not\subseteq S_{\overline{x}}$
- $S = S_x \otimes S_{\overline{x}} = \{C_x \otimes C_{\overline{x}} | C_x \in S_x and C_{\overline{x}} \in S_{\overline{x}}\}$

$$\Downarrow$$

$$\phi = \langle P(X_1, ...X_n) \backslash \{x\}, (M(X_1, ..., X_n) \cup S) \backslash (S_x \cup S_{\overline{x}})\rangle$$

The size of the formula should not increase in terms of literals

$$dim(S) \leq dim(S_x) + dim(S_{\overline{x}}) + \mathrm{K}$$

Clause Elimination by Subsumption

- Given a QBF which matrix is

$$\phi = (x_1 \vee x_2) \wedge (x_1 \vee x_2 \vee x_3)$$

$(x_1 \vee x_2)$ subsumes $(x_1 \vee x_2 \vee x_3)$

del: $(x_1 \vee x_2 \vee x_3)$

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$$\phi = (x_1 \vee x_2) \wedge (\bar{x}_1 \vee x_2 \vee x_3)$$

$$(x_1 \vee x_2) \otimes_{x_1} (\bar{x}_1 \vee x_2 \vee x_3) = (x_2 \vee x_3)$$

Q-Resolving the two clauses on x_1 results in $c = (x_2 \vee x_3)$ which subsumes $(\bar{x}_1 \vee x_2 \vee x_3)$, then $(x_1 \vee x_2)$ self-subsumes $(\bar{x}_1 \vee x_2 \vee x_3)$

add: $(x_2 \vee x_3)$
del: $(\bar{x}_1 \vee x_2 \vee x_3)$

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Advertisement

Presentation of:

Giunchiglia E., Marin. P., and Narizzano M.

sQueezeBF:

An Effective Preprocessor for QBFs Based on Equivalence Reasoning

SAT'10, Monday, h16:30 @ Appleton Tower, Room LT2

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The new qubeEngine

Search-based p-cnf QBF solver

- unit, pure & don't care literal propagation
- conflict & solution non-chronological backtracking
- recursive resolution for eliminating tautological-clauses

Data Structure

$\mathbf{W}_1$	$\mathbf{W}_2$	l_0	l_1	$\cdots$	l_n	**0**

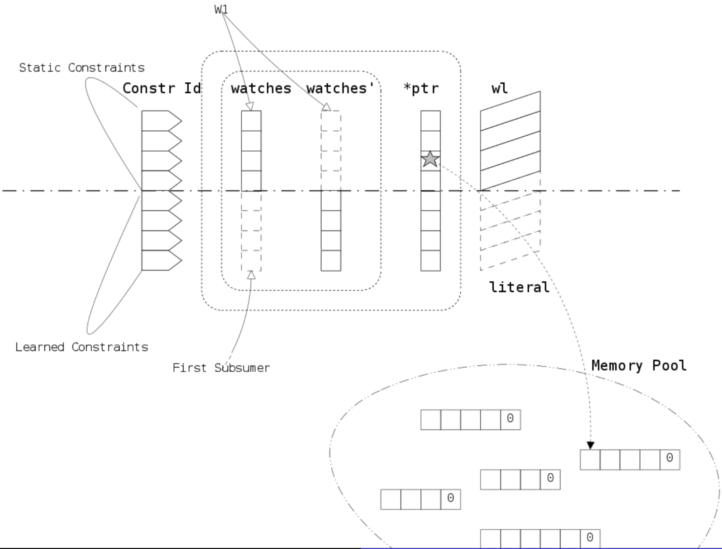
Figure: Constraint Encoding

Q	**C**	b_{28}	b_{27}	$\cdots$	b_0	**S**

Figure: Literal Encoding

- Binary and n-ary clauses are stored separately
- Main search loop designed to be lazy
- New learning algorithm

Data Structure (cont'd)



Search Loop

function qubeEngine(φ)
  $\mu = \emptyset$
  while (TRUE)
    Propagate(μ,φ)
    if (empty $\notin \varphi$)
      μ.*push*(Heuristic(φ))
    if (empty $\in \varphi$)
      Backtrack(μ,φ)
    else if (μ.*top*(*B*) $==$ 0)
      BuildPrimeImplicant(μ,φ)
      Backtrack(μ,φ)
    if (emptyClause $\in \varphi$)
      return FALSE
    if (emptyTerm $\in \varphi$)
      return TRUE

Propagation Loop

function Propagate(μ,φ)
start:
while (($l = \mu.next(B)) \neq 0$)
ResolveBinaries(l,μ,φ)
if (empty $\in \varphi$) **return**
while (($l = \mu.next(S)) \neq 0$)
Subsume(l,φ)
while (($l = \mu.next(N)) \neq 0$)
ResolveNaries(l,μ,φ)
if (empty $\in \varphi$) **return**
if ($\mu.top(B) \neq 0$) **goto** start
while (($l = \mu.next(P)) \neq 0$)
Search4pure(l,μ,φ)
if ($\mu.top(B) \neq 0$) **goto** start

Conflict and solution analysis

$$\varphi = \forall y_1 y_2 y_3 \exists x_1 x_2 \forall y_4 y_5 \exists x_3 x_4 \phi$$

$$\mu = \{y_1 ; x_1 ; y_4 ; y_2 ; y_5 ; x_4 ; y_3 ; x_3\}$$
$$b ; u ; p ; b ; u ; dc ; b ; u$$
$$@ \; 1 ; 1 ; 1 ; 2 ; 2 ; 2 ; 3 ; 3$$

$$C = x_1 \vee y_5 \vee x_3$$

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$$C = x_1 \vee y_5 \vee \neg y_5 \vee x_3$$

Conflicting Literals!

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$$C = x_1 \lor y_5 \lor \neg y_5 \lor x_3$$

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out of order resolution

Prime implicant

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5 4 3 2 1 $\Longleftarrow \forall$

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$\exists \Longrightarrow$ 6 7 8 9

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$$T = y_1 \wedge y_2 \wedge y_3 \wedge x_1 \wedge x_2 \wedge y_4 \wedge y_5 \wedge x_3 \wedge \neg x_4$$

Prime implicant

$$\varphi = \forall y_1 y_2 y_3 \exists x_1 x_2 \forall y_4 y_5 \exists x_3 x_4 \phi$$

5 4 3 2 1 $\Longleftarrow \forall$

$\exists \Longrightarrow$ 6 7 8 9

$$\mu = \{y_1 \ ; x_2 \ ; y_4 \ ; y_5 \ ; y_2 \ ; x_4 \ ; y_3 \ ; x_1 \ ; x_3\}$$

$$b \ ; u \ ; p \ ; u \ ; b \ ; dc \ ; b \ ; u \ ; p$$

$$T = y_1 \wedge y_2 \wedge y_3 \wedge x_1 \wedge x_2 \wedge y_4 \wedge y_5 \wedge x_3 \wedge \neg x_4$$

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$$\mu = \{y_1 \;;\; x_2 \;;\; y_4 \;;\; y_5 \;;\; y_2 \;;\; x_4 \;;\; y_3 \;;\; x_1 \;;\; x_3\}$$

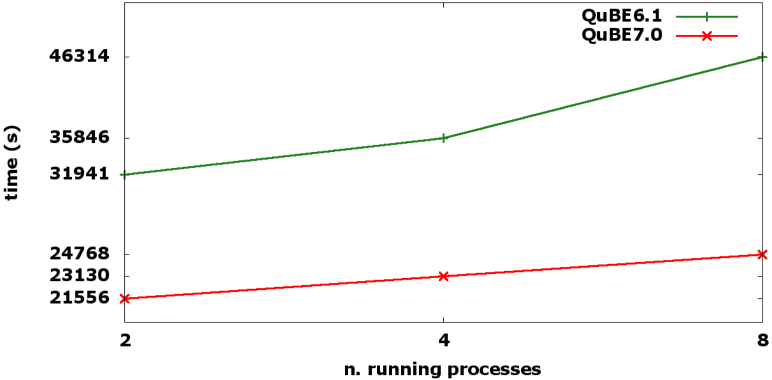
$$b \;;\; u \;;\; p \;;\; u \;;\; b \;;\; dc \;;\; b \;;\; u \;;\; p$$

$$T = y_1 \wedge y_2 \wedge y_3 \wedge x_1 \wedge x_2 \wedge y_4 \wedge y_5 \wedge x_3 \wedge \neg x_4$$

Results

instances	568 fixed-structure, selected for QBFEval'10
hardware I	farm of 9 PCs, intel Core2 Duo 2.13 GHz, 4 GB RAM, GNU Linux Debian 2.6.18.5
hardware II	cluster of 4 IBM HS21 blades, 2x intel Quad Core Xeon 2.5 GHz, 16 GB RAM, GNU Linux CentOS 5
time limit	set to 1200 s

Efficiency on new computer architectures



Results on QBFEval'10 testset

Solver	Total		Sat		Unsat	
	#	Time	#	Time	#	Time
aqme-10	434	32091.1	184	15825.6	250	16265.5
QuBE7.0	**403**	44204.3	180	26342.4	223	17861.9
depqbf	370	21515.3	164	13771.8	206	7743.5
qmaiga	361	43058.1	180	20696.6	181	22361.4
depqbf-pre	356	18995.9	172	12453.8	184	6542.1
AIGSolve	329	22786.6	171	12091.5	158	10695.1
struqs-10	240	32839.7	109	13805.5	131	19034.2
nenofex-qbfeval10	225	13786.9	109	8241.9	116	5545.1
quantor-3.1	205	6711.4	100	4130.6	105	2580.7

Comparison against QuBE6.x on QBFEval'08 testset

Solver	Total	Sat	Unsat
AQME-1NN	2434	977	1457
QuBE7.0+ps	**2367**	901	1466
QuBE7.0	**2277**	852	1425
QuBE6.1	**2144**	828	1316
Nenofex	985	459	526
quantor3.0	972	485	487
ssolve-A	965	450	515
ssolveB	960	450	510
ssolveC	939	444	495

QuBE7.0 ran on a slightly different CPU

Future Work

*QuBE*7.0 is a state-of-the-art QBF Solver

... future improvements:

sQueezeBF

- equivalence reasoning on further types of gates
- dependency schemas
- criteria for stopping

qubeEngine

- progress saving
- learning mechanism/schema
- dependency schemas
- add unlearning strategies
- parallel search

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qubeEngine

- progress saving
- learning mechanism/schema
- dependency schemas
- add unlearning strategies
- parallel search

Questions?

Thank you

Command line

Available options:

`-ss`	:	enables Selfsubsumption resolution
`-qr`	:	enables Variable Elimination by Q-Resolution
`-es`	:	enables equivalence reasoning on AND/OR gates
`-3e`	:	enables equivalence reasoning on XOR gates
`-er`	:	enables equivalence rewriting
`-all`	:	enables all the above preprocessing techniques
`-noprepro`	:	disables sQueezeBF, and gives the input formula
	:	to qubeEngine
`-solve`	:	solves by using qubeEngine

Pure and Unit Literal Detection

Unit literal

A literal l is unit in φ iff it is the only existential in some clause $c \in \phi$ and all the universal literals $l' \in c$ are s.t. $depth(l') > depth(l)$

Pure literal

An existential (resp. universal) literal l is pure in φ iff $\bar{l} \notin c$ (resp. $l \notin c$) for all clauses $c \in \phi$

Equivalence Substitution

Satisfiability-preserving transformation

$$\bar{l} \leftarrow \overline{l_1} \wedge \overline{l_2}$$
$$l \leftarrow l_1 \vee l_2$$
$$l \equiv l_1 \vee l_2$$

$$\varphi \quad (\bar{l} \vee \overline{l_1} \vee y) \wedge (l \vee \overline{l_2} \vee \overline{y}) \wedge (\bar{l} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2})$$

$$(\overline{l_1} \vee \overline{l_1} \vee y) \quad (l_1 \vee l_2 \vee \overline{l_2} \vee \overline{y})$$
$$(\overline{l_1} \vee \overline{l_2} \vee y)$$

$$\Downarrow$$
$$\varphi' = \varphi(\gamma / l)$$

$$\Downarrow$$
$$(\overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee y)$$

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$$(\overline{l_1} \vee \overline{l_1} \vee y) \quad (l_1 \vee l_2 \vee \overline{l_2} \vee \bar{y})$$
$$(\overline{l_1} \vee \overline{l_2} \vee y)$$

$$\Downarrow \qquad \Downarrow$$

$$\varphi' = \varphi(\gamma/l) \qquad (\overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee y)$$

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$$(\bar{l_1} \vee \bar{l_2} \vee y)$$

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$$\varphi' = \varphi(\gamma / l)$$

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$$(\bar{l_1} \vee y) \wedge (\bar{l_1} \vee \bar{l_2} \vee y)$$

Equivalence Substitution

Satisfiability-preserving transformation

$$\bar{l} \leftarrow \bar{l_1} \wedge \bar{l_2}$$
$$l \leftarrow l_1 \vee l_2$$
$$\overbrace{l \equiv l_1 \vee l_2}$$

$$\varphi \qquad (\bar{l} \vee \bar{l_1} \vee y) \wedge (l \vee \bar{l_2} \vee \bar{y}) \wedge \overbrace{(\bar{l} \vee l_1 \vee l_2) \wedge (l \vee \bar{l_1}) \wedge (l \vee \bar{l_2})}$$

$$\underbrace{(\bar{l} \vee \bar{l_1} \vee y)}_{(\bar{l_1} \vee \bar{l_1} \vee y)\ (\bar{l_1} \vee \bar{l_2} \vee y)} \qquad \underbrace{(l \vee \bar{l_2} \vee \bar{y})}_{(l_1 \vee l_2 \vee \bar{l_2} \vee \bar{y})}$$

$$\Downarrow \qquad \Downarrow$$

$$\varphi' = \varphi(\gamma/l) \qquad (\bar{l_1} \vee y) \wedge (\bar{l_1} \vee \bar{l_2} \vee y)$$

Dependency Graph

Given the QBF:

$$\varphi = \phi \wedge (l_1 \equiv l_2 \vee l_4) \wedge (l_2 \equiv l_3 \vee l_4) \wedge (l_3 \equiv l_1 \vee l_5)$$

if l_1 is eliminated first, then when l_3 is substituted in the formula, l_1 is reintroduced

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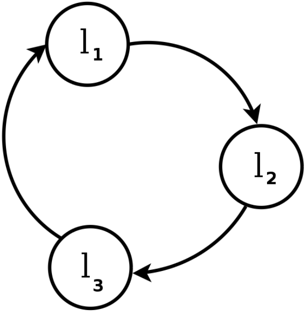

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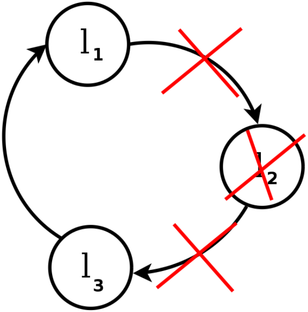

Variable Elimination by Equivalence Checking Example

$$\overline{x} \leftarrow \overline{l_1} \vee \overline{l_2}$$
$$x \leftarrow l_1 \wedge l_2$$
$$x \equiv l_1 \wedge l_2$$

1: $(x \vee \overline{l_1} \vee y) \wedge (\overline{x} \vee \overline{l_2} \vee \overline{y}) \wedge (\overline{x} \vee l_1 \vee l_2) \wedge (x \vee \overline{l_1}) \wedge (x \vee \overline{l_1})$

$(l_1 \vee \overline{l_1} \vee y)$ $\quad (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$
$(l_2 \vee \overline{l_1} \vee y)$

$\Downarrow$

2: $(l_2 \vee \overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$

Variable Elimination by Equivalence Checking Example

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$x \equiv l_1 \wedge l_2$

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$(l_1 \vee \overline{l_1} \vee y)$ $(\overline{l_1} \vee \overline{l_2} \vee \overline{y})$
$(l_2 \vee \overline{l_1} \vee y)$

$\Downarrow$

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$$(l_1 \vee \overline{l_1} \vee y) \quad (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$$
$$(l_2 \vee \overline{l_1} \vee y)$$

$$\Downarrow$$

$$2: (l_2 \vee \overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$$

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$$(l_1 \vee \overline{l_1} \vee y) \quad (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$$
$$(l_2 \vee \overline{l_1} \vee y)$$

$$\Downarrow$$

$$2: (l_2 \vee \overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$$

Variable Elimination by Equivalence Checking
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$(l_1 \vee \overline{l_1} \vee y)$ $(\overline{l_1} \vee \overline{l_2} \vee \overline{y})$
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$(l_2 \vee \overline{l_1} \vee y)$

$$\Downarrow$$

2: $(l_2 \vee \overline{l_1} \vee y) \wedge (\overline{l_1} \vee \overline{l_2} \vee \overline{y})$

Equivalence Rewriting (cont'd)

$$l \rightarrow l_1 \wedge l_2$$
$$\overline{l'} \rightarrow \overline{l_1} \vee \overline{l_2}$$
$$l \vee \overline{l'}$$
$$l \equiv l_1 \wedge l_2$$

$$(l \vee y \vee z) \wedge (\bar{l} \vee y \vee \bar{z}) \wedge (\bar{l} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2})$$

$$(l' \vee y \vee z)$$

$$\Downarrow$$

$$(l' \vee y \vee z) \wedge (\bar{l} \vee y \vee \bar{z}) \wedge (\overline{l'} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2}) \wedge (l \vee \overline{l'})$$

Equivalence Rewriting (cont'd)

$$l \rightarrow l_1 \wedge l_2$$
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$$\underbrace{}_{(l' \vee y \vee z)}$$

$$\Downarrow$$

$$(l' \vee y \vee z) \wedge (\bar{l} \vee y \vee \bar{z}) \wedge (\overline{l'} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2}) \wedge (l \vee \overline{l'})$$

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$$\underbrace{\quad}_{(l' \vee y \vee z)}$$

$$\Downarrow$$

$$(l' \vee y \vee z) \wedge (\bar{l} \vee y \vee \bar{z}) \wedge (\overline{l'} \vee l_1 \vee l_2) \wedge (l \vee \overline{l_1}) \wedge (l \vee \overline{l_2}) \wedge (l \vee \overline{l'})$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi$$

$$\alpha = \top$$

$$\varphi' = (l' \vee \beta) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \text{ (l is pure)}$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = (l \vee \alpha) \wedge (l \equiv \gamma) \wedge \phi$$

$\beta = \top$

$$\varphi' = (l \vee \alpha) \wedge (l \rightarrow \gamma) \wedge \phi \text{ } (\overline{l'} \text{ is pure})$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = (l \equiv \gamma) \wedge \phi$$

$\alpha = \top$ and $\beta = \top$

$$\varphi' = \phi \ (l \text{ and } \overline{l'} \text{ are pure})$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \lor \alpha) \land (\bar{l} \lor \beta) \land (l \equiv \gamma) \land \phi \text{ *original*}$$

$$\Downarrow$$

$$\varphi = \beta \land \gamma \land \phi$$

$\alpha = \bot$ (*l* is unit)

$$\varphi' = (l' \lor \beta) \land \gamma \land (\gamma \rightarrow \overline{l'}) \land \phi$$

$$\Uparrow$$

$$\varphi' = (l \lor \alpha) \land (l' \lor \beta) \land (l \rightarrow \gamma) \land (\gamma \rightarrow \overline{l'}) \land \phi \text{ *rewritten*}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \text{ original}$$

$$\Downarrow$$

$$\varphi = \beta \wedge \gamma \wedge \phi$$

$\alpha = \bot$ (*l* is unit)

$$\varphi' = (l' \vee \beta) \wedge \gamma \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi$$

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Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = \alpha \wedge (\neg\gamma) \wedge \phi$$

$$\beta = \bot$$

$$\varphi' = (l \vee \alpha) \wedge (l \rightarrow \gamma) \wedge (\neg\gamma) \wedge \phi \text{ (l' is unit)}$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = \alpha \wedge (\neg\gamma) \wedge \phi$$

$\beta = \bot$

$$\varphi' = (l \vee \alpha) \wedge (l \rightarrow \gamma) \wedge (\neg\gamma) \wedge \phi \text{ (l' is unit)}$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Influence of Equivalence Rewriting

$$\varphi = (l \vee \alpha) \wedge (\bar{l} \vee \beta) \wedge (l \equiv \gamma) \wedge \phi \textit{ original}$$

$$\Downarrow$$

$$\varphi = \alpha \wedge (\neg\gamma) \wedge \phi$$

$\beta = \perp$

$$\varphi' = (l \vee \alpha) \wedge (l \rightarrow \gamma) \wedge (\neg\gamma) \wedge (\bar{l} \vee \overline{l'}) \wedge \phi \ (l' \text{ is unit})$$

$$\Uparrow$$

$$\varphi' = (l \vee \alpha) \wedge (l' \vee \beta) \wedge (l \rightarrow \gamma) \wedge (\gamma \rightarrow \overline{l'}) \wedge \phi \textit{ rewritten}$$

Example:

Given

$$\exists x_1 \forall y \exists x_2\, x_3 (\bar{x}_1 \vee y \vee \bar{x}_2) \wedge (x_1 \vee y \vee x_3) \wedge (y \vee \bar{x}_3) \wedge (x_2 \vee x_3) \wedge (\bar{x}_2 \vee \bar{x}_3)$$

let's remove x_3 by Q-Resolution:

$$\begin{array}{ll} (x_1 \vee y \vee x_3) \otimes (y \vee \bar{x}_3) & \Rightarrow x_1 \\ (x_1 \vee y \vee x_3) \otimes (\bar{x}_2 \vee \bar{x}_3) & \Rightarrow x_1 \vee y \vee \bar{x}_2 \\ (x_2 \vee x_3) \otimes (y \vee \bar{x}_3) & \Rightarrow y \vee x_2 \\ (x_2 \vee x_3) \otimes (\bar{x}_2 \vee \bar{x}_3) & \Rightarrow x_2 \vee \bar{x}_2 \Rightarrow \top \end{array}$$

The given QBF becomes:

$$\exists x_1 \forall y \exists x_2 (\bar{x}_1 \vee y \vee \bar{x}_2) \wedge (x_1) \wedge (x_1 \vee y \vee \bar{x}_2) \wedge (y \vee x_2)$$

Variable Elimination by Q-Resolution Example:

Given

$$\exists x_1 \forall y \exists x_2\, x_3 (\bar{x}_1 \vee y \vee \bar{x}_2) \wedge (x_1 \vee y \vee x_3) \wedge (y \vee \bar{x}_3) \wedge (x_2 \vee x_3) \wedge (\bar{x}_2 \vee \bar{x}_3)$$

let's remove x_3 by Q-Resolution:

$$\begin{array}{ll} (x_1 \vee y \vee x_3) \otimes (y \vee \bar{x}_3) & \Rightarrow x_1 \\ (x_1 \vee y \vee x_3) \otimes (\bar{x}_2 \vee \bar{x}_3) & \Rightarrow x_1 \vee y \vee \bar{x}_2 \\ (x_2 \vee x_3) \otimes (y \vee \bar{x}_3) & \Rightarrow y \vee x_2 \\ (x_2 \vee x_3) \otimes (\bar{x}_2 \vee \bar{x}_3) & \Rightarrow x_2 \vee \bar{x}_2 \Rightarrow \top \end{array}$$

The given QBF becomes:

$$\exists x_1 \forall y \exists x_2 (\bar{x}_1 \vee y \vee \bar{x}_2) \wedge (x_1) \wedge (x_1 \vee y \vee \bar{x}_2) \wedge (y \vee x_2)$$

Backjumping

Problem

Time spent visiting parts of the search space in vain because some choices may not be responsible for the result of the search

Solution [Giunchiglia, Narizzano, Tacchella 2001]

1. for each node of the search tree, compute a subset (called "reason") of the assigned variables which are responsible for the current result; and
2. while backtracking, skip nodes which do not belong to the reason for the discovered conflicts/solutions:

   CBJ Conflict Backjumping
   SBJ Solution Backjumping

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   CBJ Conflict Backjumping
   SBJ Solution Backjumping

Example

Original Formula:

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_4 x_5$$
$$(x_2 \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_2} \vee x_4) \wedge (\overline{x_3} \vee x_4) \wedge (x_1 \vee y_1 \vee x_3 \vee x_5) \wedge$$
$$(\overline{x_3} \vee x_4 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_4) \wedge (x_1 \vee y_2 \vee \overline{x_4}) \wedge$$
$$(\overline{y_1} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

$(\overline{x_3} \vee x_4 \vee x_5)$ is subsumed;

$x_4 \equiv x_3 \vee x_2$: remove x_4 by Equivalence Checking:

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_5$$
$$(x_1 \vee y_1 \vee x_3 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_3}) \wedge (x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

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$$(x_1 \vee y_1 \vee x_3 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_3}) \wedge (x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Example

Original Formula:

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_4 x_5$$
$$(x_2 \lor x_3 \lor \overline{x_4}) \land (\overline{x_2} \lor x_4) \land (\overline{x_3} \lor x_4) \land (x_1 \lor y_1 \lor x_3 \lor x_5) \land$$
$$(\overline{x_3} \lor x_4 \lor x_5) \land (x_1 \lor y_1 \lor x_2 \lor \overline{x_5}) \land (x_1 \lor \overline{y_2} \lor x_4) \land (x_1 \lor y_2 \lor \overline{x_4}) \land$$
$$(\overline{y_1} \lor \overline{x_3}) \land (\overline{x_1} \lor x_2 \lor \overline{x_3})$$

$(\overline{x_3} \lor x_4 \lor x_5)$ is subsumed;

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$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_5$$
$$(x_1 \lor y_1 \lor x_3 \lor x_5) \land (x_1 \lor y_1 \lor x_2 \lor \overline{x_5}) \land (x_1 \lor \overline{y_2} \lor x_2 \lor x_3) \land$$
$$(x_1 \lor y_2 \lor \overline{x_3}) \land (x_1 \lor y_2 \lor \overline{x_2}) \land (\overline{x_1} \lor x_2 \lor \overline{x_3})$$

Example (cont.)

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_5$$
$$(x_1 \vee y_1 \vee x_3 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_3}) \wedge (x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Try to remove x_2 by Q-Resolution:

$$(x_1 \vee \overline{y_2} \vee x_2 \vee x_3\} \otimes_{x_2} \{x_1 \vee y_2 \vee \overline{x_2}\} = \{x_1 \vee y_2 \vee \overline{y_2} \vee x_3)$$

y_2 locks (... same x_3 ...)

Try to remove x_5 by Q-Resolution:

$$(x_1 \vee y_1 \vee x_3 \vee x_5) \otimes_{x_5} (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) = (x_1 \vee y_1 \vee x_2 \vee x_3)$$

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3$$
$$(x_1 \vee y_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge (x_1 \vee y_2 \vee \overline{x_3}) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Example (cont.)

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_5$$
$$(x_1 \vee y_1 \vee x_3 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_3}) \wedge (x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

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y_2 locks (... same x_3 ...)

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$$(x_1 \vee y_1 \vee x_3 \vee x_5) \otimes_{x_5} (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) = (x_1 \vee y_1 \vee x_2 \vee x_3)$$

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3$$
$$(x_1 \vee y_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge (x_1 \vee y_2 \vee \overline{x_3}) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Example (cont.)

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3 x_5$$
$$(x_1 \vee y_1 \vee x_3 \vee x_5) \wedge (x_1 \vee y_1 \vee x_2 \vee \overline{x_5}) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge$$
$$(x_1 \vee y_2 \vee \overline{x_3}) \wedge (x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Try to remove x_2 by Q-Resolution:

$$(x_1 \vee \overline{y_2} \vee x_2 \vee x_3\} \otimes_{x_2} \{x_1 \vee y_2 \vee \overline{x_2}\} = \{x_1 \vee y_2 \vee \overline{y_2} \vee x_3)$$

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Example (cont.)

$\exists x_1 \forall y_1 y_2 \exists x_2 x_3$
$(x_1 \vee y_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge (x_1 \vee y_2 \vee \overline{x_3}) \wedge$
$(x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$

y_1 is pure:

$\exists x_1 \forall y_2 \exists x_2 x_3$
$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge (x_1 \vee y_2 \vee \overline{x_3}) \wedge$
$(x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$

Example (cont.)

$$\exists x_1 \forall y_1 y_2 \exists x_2 x_3$$
$$(x_1 \vee y_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{y_2} \vee x_2 \vee x_3) \wedge (x_1 \vee y_2 \vee \overline{x_3}) \wedge$$
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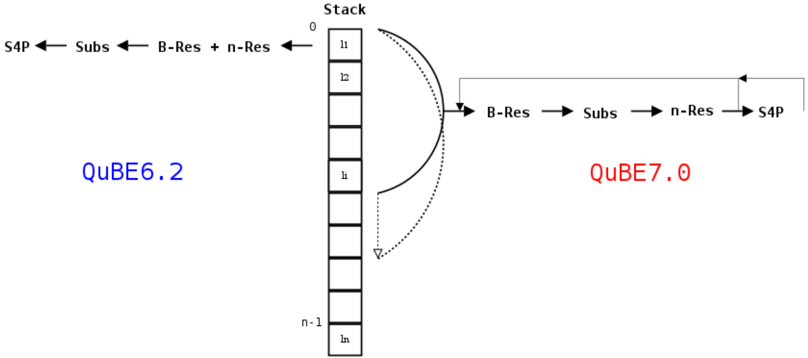
$$(x_1 \vee y_2 \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$$

Main Search Loop

$\forall l \in$ AssignmentStack we apply 4 different operations:

1. B–Res: Resolution (a.k.a. *Unit Propagation*) on Binary Constraints
2. Subs: Subsumption of Binary and *n*-ary Constraints
3. *n*-Res: Resolution on *n*-ary Constraints
4. S4P: Search for Pure Literals

Comparison of QuBE6.x a and QuBE7 main loops



Results on QBFEval'10 testset

Solver	Total		Sat		Unsat	
	#	Time	#	Time	#	Time
aqme-10	434	32091.1	184	15825.6	250	16265.5
QuBE7	**410**	52142.1	189	35489.7	221	1402.62
QuBE7.0	**403**	44204.3	180	26342.4	223	17861.9
depqbf	370	21515.3	164	13771.8	206	7743.5
qmaiga	361	43058.1	180	20696.6	181	22361.4
depqbf-pre	356	18995.9	172	12453.8	184	6542.1
AIGSolve	329	22786.6	171	12091.5	158	10695.1
struqs-10	240	32839.7	109	13805.5	131	19034.2
nenofex-qbfeval10	225	13786.9	109	8241.9	116	5545.1
quantor-3.1	205	6711.4	100	4130.6	105	2580.7